CHEMISTRY PAPER-I
PART I –CHEMISTRY
SECTION –I (TOTAL MARKS: 21)
(SINGLE CORRECT CHOICE TYPE)

This section contains 7 multiple choice questions. Each question has four choices (a),(b), (c) and (d) out of which **ONLY ONE** is correct.

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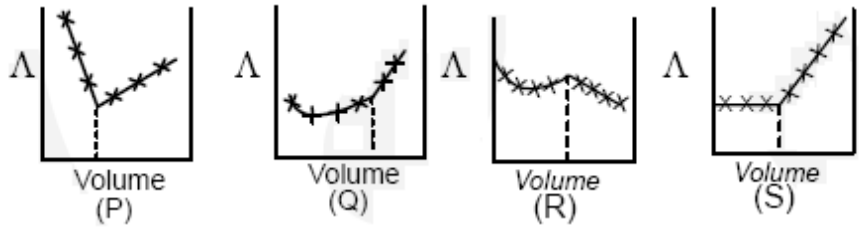
1. Dissolving 120g of urea (mol. wt 60) in 1000g of water gave a solution of density 1.15g/mL. The molarity of the solution is
   (a) 1.78 M (b) 2.00 M (c) 2.05 M (d) 2.22 M]

1. **(c)**

2. $AgNO_3$(aq.) was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance $(\wedge)$ versus the volume of $AgNO_3$ is



   (a) (P) (b) (Q) (c) (R) (d) (S)

2. **(d)** $AgNO_3 + KCl \rightarrow AgCl\downarrow + KNO_3$
   Conductivity will remain same for some time then gradually increase due to excess of KCl.

3. Among the following compounds, the most acidic is
   (a) p-nitrophenol (b) p-hydroxybenzoic zcid
   (c) ohydroxybenzoic acid (d) p-toluic acid

3. **(c)** Due to ortho effect

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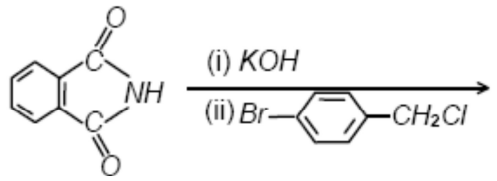
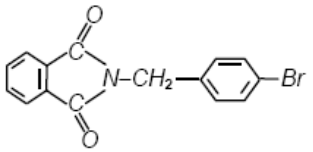
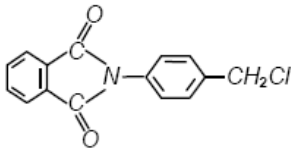
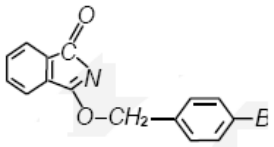
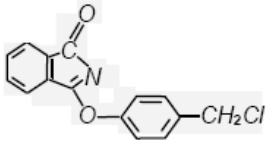
4. The major product of the following reaction is



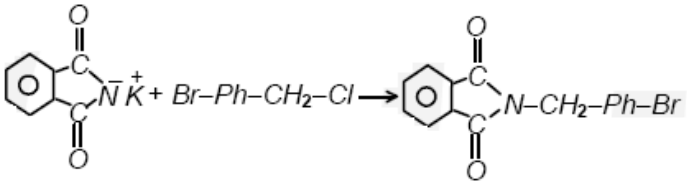
(a) 

(b) 

(c) 

(d) 

4. **(a)**



5. Extra pure N_2 can be obtained by heating

(a) NH_3 with CuO
(b) NH_4NO_3
(c) $(NH_4)_2Cr_2O_7$
(d) $Ba(N_3)_2$

5. **(d)** $Ba(N_3)_2 \xrightarrow{\Delta H} Ba \downarrow + 3N_2$

6. Geometrical shapes of the complexes formed by the reaction of Ni^{2+} with Cl^-, CN^- and H_2O respectively are

(a) Octahedral, tetrahedral and square planar
(b) Tetrahedral, square planar and octahedral
(c) Square planar, tetrahedral and octahedral
(d) Octahedral, square planar and octahedral

6. **(b)** $[NiCl_4]^{-2}$ sp^3

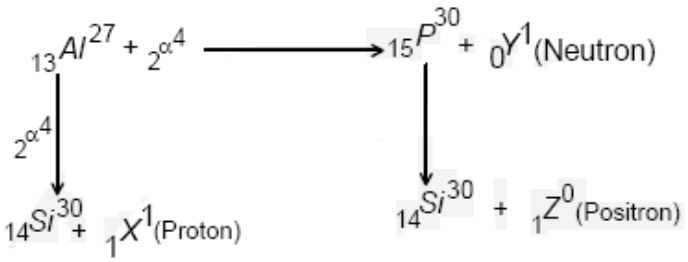
$[Ni(CN)_4]^{-2}$ dsp^2

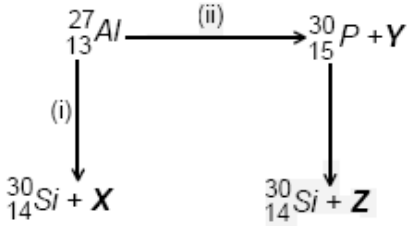
$[Ni(H_2O)_6]^{+2}$ sp^3d^2

7. Bombardment of aluminium by $\alpha -$ particle leads to its disintegration in two ways, (i) and (ii) as shown. Products X,Y and Z respectively are,

(a) Proton, neutron, positron
(b) Neutron, positron, proton
(c) Proton, Positron, neutron
(d) proton, neutron

7. **(a)**





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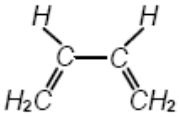
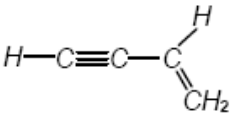
SECTION –II (TOTAL MARKS: 16)
(MULTIPLE CORRECT ANSWER TYPE)

This section contains 4 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **One OR MORE** may be correct.

8. Amongst the given options, the compound(s) in which all the atoms are in one plane in all the possible conformations (if any), is (are)

(a) 

(b) 

(c) $H_2C=C=O$

(d) $H_2C=C=CH_2$

8. **(a,b,c)** Taking only stable conformers in account

9. According to kinetic theory of gases
(a) Collisions are always elastic
(b) Heavirer molecules tarfer more momentum to the wall of the container.
(c) Only a small number of molecules have very high velocity
(d) Between collisions, the molecules move in straight lines with constant velocities.

9. **(a,d)** Option c is correct but not mentioned in the kinetic theory of gases

10. The correct statement(s) pertaining to the adsorption of a gas on a solid surface is (are)
(a) Adsorption is always exothermic
(b) Physisorption mat transform into chemisorptions at high temperature.
(c) Physiosorption increases with increasing temperature but chemisorptions decreases with increasing temperature.
(d) Chemisorptions is more exothermic than physisorption, however it is very slow due to higher energy of activation.

10. **(abd)** Physisorption

11. Extraction of metal from the ore cassiterite involves
(a) Carbon reduction of an oxide ore
(b) self-reduction of a sulphide ore
(c) Removal of copper impurity
(d) Removal of iron impurity

11. **(acd)** $SnO_2 + 2C \rightarrow Sn + 2CO$
Tin is purified by liquation and polling

SECTION-III (TOTAL MARKS 15)
(PARAGRAPH TYPE)

This section contains **2 paragraphs**. Based upon one of the paragraphs 2 multiple choice questions and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Questions Nos. 12 and 13

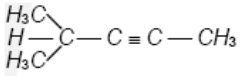
An acyclic hydrocarbon P, having molecular C_6H_{10}, gave acetone as the only organic product through the following sequence of reactions, in which Q is an intermediate organic compound.

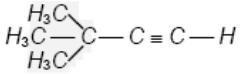
$$\underset{(C_6H_{10})}{P} \xrightarrow[\text{(ii) } NaBH_4 \text{ / ethanol; (iii) dil. acid}]{\text{(i) dil. } H_2SO_4 / HgSO_4} Q \xrightarrow[\text{(ii) } O_3;\ \text{(iii) } Zn / H_2O]{\text{(i) conc. } H_2SO_4 \text{ (catalytic amount) } (-H_2O)} 2$$

12. The structure of compound P is

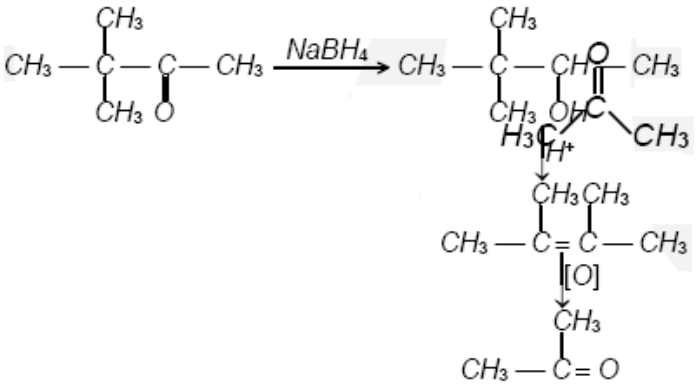
(a) $CH_3CH_2CH_2CH_2 - C \equiv C - H$

(b) $H_3CH_2C - C \equiv C - CH_3CH_3$

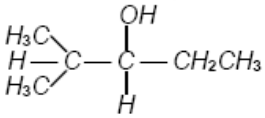
(c) 

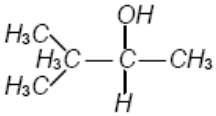
(d) 

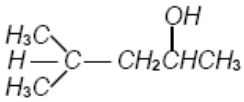
12. **(d)**



13. The structure of the compound Q is

(a) 

(b) 

(c) 

(d) $CH_3CH_2CH_2CH(OH)CH_2CH_3$

13. **(b)**

Paragraph for Questions Nos. 14 and 16

When a metal rod M is dipped into an aqueous colourless concentrated solution of compound N, the solution turns light blue. Addition of aqueous NaCl to the blue solution gives a white precipitate O. Addition of aqueous NH_3 dissolves O and gives an intense blue solution.

14. The metal rod M is
(a) Fe (b) Cu (c) Ni (d) Co

14. **(b)**

$$Cu + 2AgNO_3 \rightarrow 2Ag + CU(No_3)_2$$
$$Ag^+ + Cl^- \rightarrow AgCl$$
$$AgCl + 2NH_3 \rightarrow Ag(NH_3)_2^+$$
$$Cu^{+2} + 4NH_3 \rightarrow Cu(NH_3)_4^+$$

15. The compound N is
(a) $AgNO_3$ (b) $An(NO_3)_2$ (c) $Al(NO_3)_3$ (d) $Pb(NO_3)_2$

15. **(a)**

16. The final solution contains
(a) $[Pb(NH_3)_4]^{2+}$ and $[CoCl_4]^{2-}$ (b) $[Al(NH_3)_4]^{3+}$ and $[Cu(NH_3)_4]^{2+}$
(c) $[Ag(NH_3)_2]^+$ and $[Cu(NH_3)4]^{2+}$ (d) $[Ag(NH_3)_2]^+$ and $[Ni(NH_3)_6]^{2+}$

16. **(c)**

SECTION –IV (TOTAL MARKS: 28)
(INTEGER ANSWER TYPE)

This section contains **7 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

17. The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300nm was wavelength falls on the metals is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
$(\phi)(\text{eV})$	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

17. **(4)** $3000A^{o} \approx 4.1$ eV.

$\therefore$ All below this value will eject photo electron

18. To an evacuated vessel with movable piston under external pressure of 1atm, 0.1mol of He and 1.0 mol of an unknown compound (vapour pressure 0.68 atm. At $0^{o}C$) are introduced. Considering the ideal gas behaviour, the total volume (in lite) of the gases at $0^{o}C$ is close to.

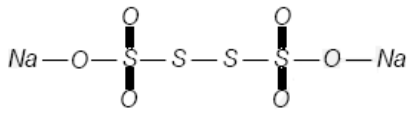
18. **(7)** $0.1 \times \frac{1}{12} \times \frac{273}{0.32} = 7$

19. Reaction of Br_2 with Na_2CO_3 in aqueous solution gives sodium bromide and sodium bromated with evolution of CO_2 gas. The number o sodium bromide molecules involved in the balanced chemical equation is

19. **(5)** $3Br_2 + 3Na_2CO_3 \rightarrow 5NaBr + NaBrO_3 + 3CO_2$

20. The difference in the oxidation numbers of the two types of sulphur atoms in $Na_2S_4O_6$ is

20. **(5)**



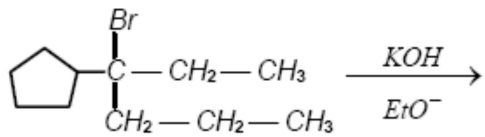
5 – 0 = 5

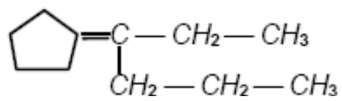
21. A decapeptide (Mol. Wt. 796) on complete hydrolysis gives hlycine (Mol. Wt 75), alanine and phenylalanine. Glycine contributes 47.0% to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is.

21. **(5)** $\frac{0.47 \times 796}{75} \approx 5$

22. The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentahylhexane using alcoholic KOH is

22. **(5)**





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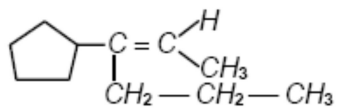
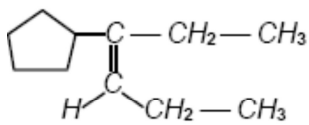


2 Cis & Trans



2 Cis & Trans

23. The maximum number of electrons that can have principal quantum number, $m_s = -1/2$ is

23. **(9)** 3s =1

PHYSICS PAPER - I
PART-II- PHYSICS
SECTION-I (TOTAL MARKS: 21)
(SINGLE CORRECT ANSWER TYPE)

This section contains **7 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

24. The wavelength of the first spectral line in the Balmer series of Hydrogen atom is 6561 $\mathring{A}$. The wavelength of the second spectral line in the Balmer series of singly – ionized helium atom is:

(a) 1215 $\mathring{A}$ (b) 1640 $\mathring{A}$ (c) 2430 $\mathring{A}$ (d) 4687 $\mathring{A}$

24. **(a)** For first spectral line in the Balmer series

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \qquad z = 1$$

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right) = R\left(5/36\right) \qquad \ldots(1)$$

For second spectral line in the Balmer series of singly – ionized helium

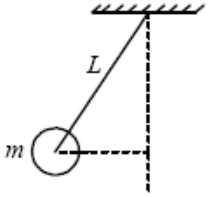
$$\frac{1}{\lambda'} = R\left(4\right)\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$= R \cdot \frac{4}{4}\left(1 - \frac{1}{4}\right) = R\left(3/4\right) \qquad \ldots(2)$$

$$\Rightarrow \left(\lambda^1/\lambda\right) = \left[\left(5\times4\right)/\left(36\times3\right)\right]$$

$$\lambda^1 = \left[5\lambda/\left(9\times3\right)\right] = \left[\left(5\times6561\right)/\left(9\times3\right)\right] = 1215\,\mathring{A}$$

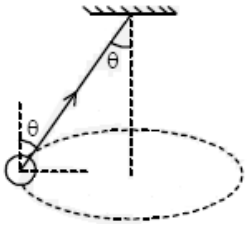
25. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is:



(a) 9 (b) 18 (c) 27 (d) 36’

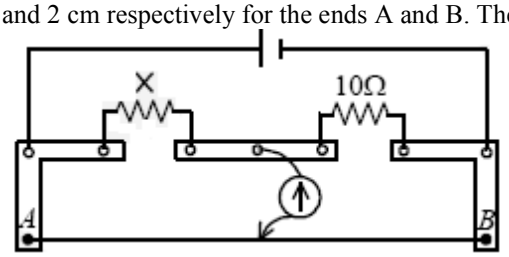
25. **(d)** $T\sin\theta = m\omega^2 r = m\omega^2 L\sin\theta$

$T = m\omega^2 L$



$$\omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324}{0.5\times0.5}} = \left(18/0.5\right) = 36$$

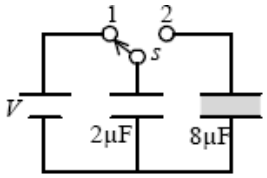
26. A meter bridge is set – up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping – key is at 52 cm. mark. The end – corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is



(a) 10.2 ohm (b) 10.6 ohm (c) 10.8 ohm (d) 11.1 ohm

26. **(b)** $(x/10) = [(52+1)/(48+2)] \Rightarrow x = 10.6\Omega$

27. A $2\,\mu F$ capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(a) 0% (b) 20% (c) 75% (d) 80%

27. **(d)** Initial energy stored in capacitor $C_1 = (Q^2/2C_1) = (Q^2/4)$

Where $Q = C_1V$

let $C_1 = 2\mu F$ $C_2 = 8\mu F$

Finally charge on $C_1 = (Q/5)$

Energy on $C_1 = [Q^2/50C_1] = (Q^2/100)$

Energy on $C_2 = (16Q^2/25) = (16Q^2/25)$

Energy dissipated, $\Delta E = \frac{Q^2}{4} - \left(\frac{Q^2}{100} + \frac{Q^2}{25}\right) = [(Q^2/4) - (Q^2/20)] = (Q^2/5)$

$\therefore$ % of its stored energy dissipated $= \left(\frac{Q^2 \times 4}{5 \times Q^2} \times 100\right) = 80\%$

28. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 ms^{-1}. The frequency of the siren heard by the car driver is:

(a) 8.50 kHz (b) 8.25 kHz (c) 7.75 kHz (d) 7.50 kHz

28. **(a)** Velocity of sound, C = 320 ms^{-1}

frequency of the siren heared by the car driver

$f = f_0[(C+V)/(C-V)] = 8[(320+10)/(320-10)] = 8.50\text{kHz}$

29. 5.6 litre of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is

(a) $(9/8)RT$ (b) $(3/2)RT$ (c) $(25/8)RT$ (d) $(9/2)RT$

29. **(a)** No. of moles, n = $(5.6/22.4) = (1/4)$

Initial Temperature = T_1

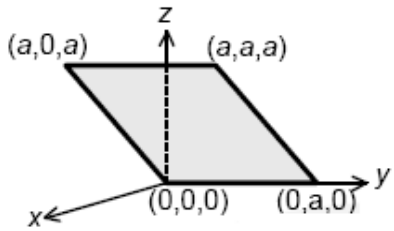
Let final temperature = T_2

$\Rightarrow T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_2 = T_1(V_1/V_2)^{\gamma-1} = T(5.6/0.7)^{5/3-1} = 4T_1$

Workdone = $\left[\left(nR\Delta T/(\gamma-1)\right)\right] = (9/8)RT_1$

30. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is:



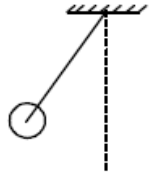
(a) $2E_0a^2$ (b) $\sqrt{2}E_0a^2$ (c) E_0a^2 (d) $\left(E_0a^2/\sqrt{2}\right)$

30. **(c)** $\phi = \left(E_0\hat{i}\right)\cdot\left(a\hat{j}\times\left(a\hat{k}-a\hat{i}\right)\right) = E_0a^2$

SECTION-II (TOTAL MARKS: 16)
(MULTIPLE CORRECT CHOICE TYPE)

This section contains **4 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONE OR MORE** may be correct.

31. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (< L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A) The disc is not free to rotate about its center and (case B) the disc is free to rotate about it center. The rod – disc system performs SHM in vertical plane after being released from the same displace position. Which of the following statement (s) is (are) true?



(a) Restoring torque in case A = Restoring torque in case B
(b) Restoring torque in case A < Restoring torque in case B
(c) Angular frequency for case a > Angular frequency for case B
(d) Angular frequency for case A < Angular frequency for case B

31. **(a, d)** Torque due to weight is same in both case $\Rightarrow$ (A)

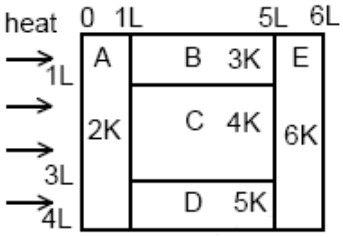
$T \propto \sqrt{(I_{sys})} \Rightarrow \omega \propto \left[1/\sqrt{I_{sys}}\right]$

Case A: When disc is not rotating then M.I of system about point of suspension = I_A
Case B : When disc is rotating then M.I. of system about point of suspension = I_B

$\because I_B = I_{rod}^{pivot} + I_{disc}^{pivot} \quad I_A = I_{rod} + \left(I_{disc}^{Pivot} + \frac{MR^2}{2}\right)$

$\because I_B < I_A \Rightarrow \omega_A < \omega_B \Rightarrow (D)$

32. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length , L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state



(a) Heat flow through A and E slabs are same
(b) Heat flow through slab E is maximum
(c) Temperature difference across slab E is smallest
(d) Heat flow through C = heat flow through B + heat flow through D

32. **(abcd)** A, (BCD), E in series. Thus $H_A = H_E = H_B + H_C + H_D \Rightarrow (A),(B)$

For parallel slabs B, C, D; $H \propto$ KA. $\therefore H_B : H_C : H_D = 3KL : 8KL : 5KL. \Rightarrow (D)$

For series, $K\Delta\theta$ is same. Since K_E is largest, $\Delta\theta_E$ is least $\Rightarrow (C)$

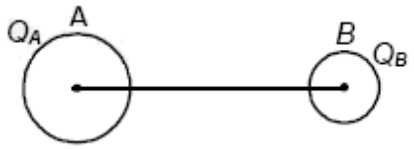
33. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi – infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement (s) is /are true?
(a) They will never come out of the magnetic field region
(b) They will come out travelling along parallel paths
(c) They will come out at the same thing
(d) They will come out at different times

33. **(b, d)** $R_P = \left(m_P V / q_p B\right)$ $R_e = m_e V / q_e B$ $\because m_p > m_e$ $\Rightarrow R_p > R_e$

$T_p = \pi m_p / qB$ $T_e = \pi m_e / qB$ $\Rightarrow T_p > T_e$ $\Rightarrow (D)$

34. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B\left(< R_A\right)$ are kept for apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then
(a) $E_A^{inside} = 0$
(b) $Q_A > Q_B$
(c) $\left(\sigma_A / \sigma_B\right) = \left(R_B / R_A\right)$
(d) $E_A^{on\ surface} < E_B^{on\ surface}$

34. **(a,b,c,d)** After redistribution



Since potential of both sphere will be same $\dfrac{Q_A}{4\pi\varepsilon_0 R_A} = \dfrac{Q_B}{4\pi\varepsilon_0 R_B} \Rightarrow Q_A > Q_B \Rightarrow (B)$

$E_A^{inside} = 0$(from Gauss law) $\Rightarrow (A)$

$\sigma_A = \dfrac{Q_A}{4\pi R_A^2}, \sigma_B = \dfrac{Q_B}{4\pi R_B^2} \Rightarrow \dfrac{\sigma_A}{\sigma_B} = \dfrac{Q_A}{Q_B} \cdot \left(\dfrac{R_B}{R_A}\right)^2 = \dfrac{R_B}{R_A} \Rightarrow (C)$

$E_A^{on\ surface} \propto \sigma_A$ & $E_B^{on\ surface} \propto \sigma_B$

$\Rightarrow E_A^{on\ surface} < E_B^{on\ surface} \Rightarrow (D)$

SECTION-III (TOTAL MARKS: 15)
(PARAGRAPH TYPE)

This section contains **2 paragraphs**. Based upon the first paragraph 2 multiple choice questions and based on the paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for questions 35 to 36

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displace relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω appropriate ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

35. Taking the electronic charge as 'e' and the permittivity as 'ε_0', use dimensional analysis to determine the correct expression for ω_p.

(a) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\frac{m\varepsilon_0}{Ne}}$ (c) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\frac{m\varepsilon_0}{Ne^2}}$

35. **(c)** Let $\omega_p \alpha \left(Ne^2\right)^a \left(m\varepsilon_0\right)^b$

$$\Rightarrow T^{-1} = \left(Ne^2\right)^a \left(m\varepsilon_0\right)^b$$

$$= \left[L^{-3}Q^2\right]^a \left[MM^{-1}L^{-3}T^2Q^2\right]^b$$

$$= L^{-3a} \quad Q^{2a} \quad L^{-3b} \quad T^{2b} \quad Q^{2b}$$

$$= L^{-3(a+b)} \quad Q^{2(a+b)} \quad T^{2b}$$

$$\Rightarrow a + b = 0 \text{ and } 2b = -1$$

$$\Rightarrow b = -(1/2) \quad \Rightarrow a = -b = (1/2)$$

$$\omega_p \propto \left(Ne^2\right)^{1/2} \left(m\varepsilon_0\right)^{-1/2}$$

$$\Rightarrow \omega_p \propto \sqrt{\frac{Ne^2}{m\varepsilon_0}}$$

36. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4\times10^{27}\,m^{-3}$. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

(a) 800 nm (b) 600 nm (c) 300 nm (d) 200 nm

36. **(b)** $f = (\omega / 2\pi)$

$$\omega = \sqrt{\frac{4\times10^{27}\times(1.6\times10^{-19})^2}{10^{-30}\times10^{-11}}}$$

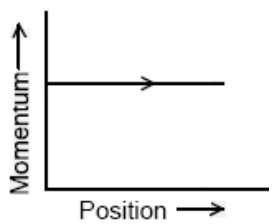
$$\omega = \sqrt{(1.6\times10^{-19})^2\times4\times10^{68}} = 2\times1.6\times10^{15} = 3.2\times10^{15}$$

$$\Rightarrow \lambda = (c/f) = \frac{3\times10^8}{\frac{3.2\times10^{15}}{2\pi}} = \frac{6\pi}{3.2}\times10^{-7}\,m$$

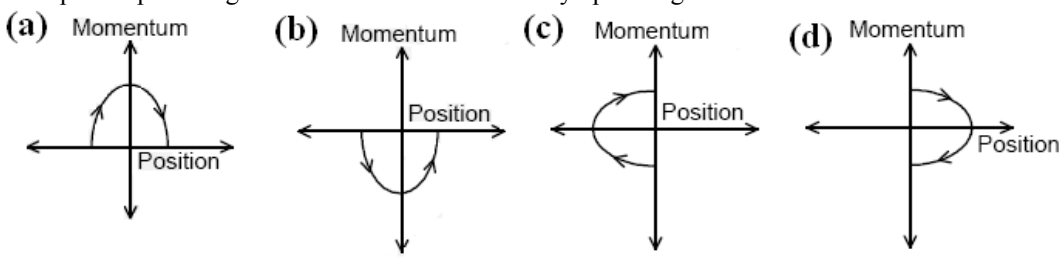
$$\Rightarrow \lambda = (600\pi/3.2)\,nm \approx 600nm$$

Paragraph for questions 37 to 39

Phase space diagrams are useful tools in analyzing all kinds of dynamic problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one – dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We used the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.

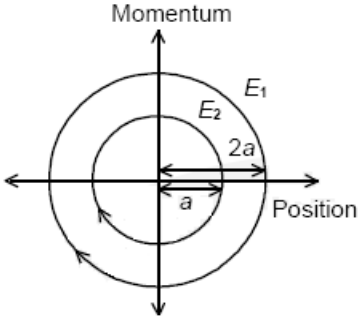


37. The phase space diagram for a ball thrown vertically up from ground is



37. **(d)** For ball thrown up, initially momentum has maximum positive value and position is zero. Then as ball moves up, the momentum decreases & position increases. At highest position, momentum is zero, position is maximum. Then ball returns during which position decreases and momentum increases in negative.

38. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then.

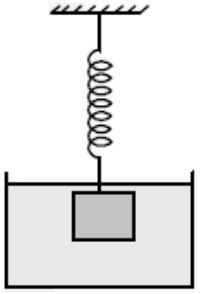


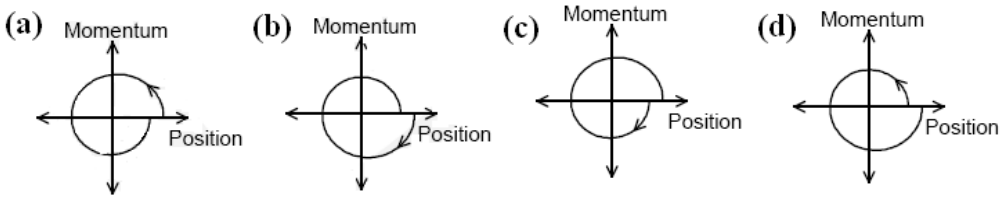
(a) $E_1 = \sqrt{2}E_2$ (b) $E_1 = 2E_2$ (c) $E_1 = 4E_2$ (d) $E_1 = 16E_2$

38. **(c)** $(E_1 / E_2) = (2a / a)^2 = 4$

$\Rightarrow E_1 = 4E_2$

39. Consider the spring – mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is:





39. **(b)** The mass has zero momentum at highest position. From here, it moves down, during which position decreases but momentum increases in negative value . After one cycle, its position is slightly less than initial value (damped oscillation).

SECTION –IV (TOTAL MARKS: 28)
(INTEGER ANSWER TYPE)

This section contains **7 questions**. The answer to each question is a single digit integer ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

40. Steel wire of length 'L' at 40^oC is suspended from the ceiling and then a mass 'm' us hung from its free end. The wire is cooled down from 40^oC to 30^oC to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is $10^{-5}/^oC$, Young's modulus of steel is $10^{11}Nm^{-2}$ and radius of the wire is 1 mm. Assume that L >> diameter of the wire. Then the valur of 'm' in kg is nearly.

40. **(3)** $(\Delta L/L)$: Same $\Rightarrow (mg/YA) = \alpha\Delta\theta$

$\Rightarrow m = [(\alpha\Delta\theta.YA)/g] = \pi \text{ kg} \approx 3 \text{ kg}$

41. The acivity of a freshly prepared radioactive sample is 10^{10} disintegration per second, whose mean life is 10^9 s. The mass j of the steel is 10^{-25} kg . The mass (in mg) of the radioactive sample is.

41. **(1)** $(1/\lambda) = 10^9 s,\ A_0 = 10^{10}$

$\Rightarrow \lambda N_0 = -10^{10}$

$\Rightarrow 10^{10} = [\lambda.(m/M)] = [m/(10^9 \times 10^{-25})]$

$\Rightarrow m = 10^{-6} kg = 1 \text{ mg}$

42. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10 μ, then N is

42. **(5)**

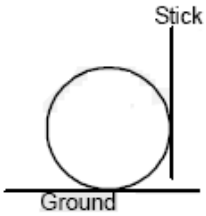
$F_u = mg(\sin\theta + \mu\cos\theta)$

$F_d = mg(\sin\theta - \mu\cos\theta)$ & $F_u = 3F_d$

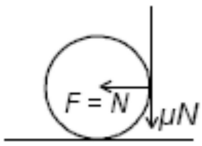
$\Rightarrow \mu = (1/2)\tan\theta = (1/2)$

$\Rightarrow N = 10\mu = 5$

43. A boy is pushing a ring of mass 2 kg and radius 0.5m with a sticks as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of $0.3ms^{-2}$. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is.



43. **(4)** $NR - \mu NR = 2mR^2\alpha$



$\Rightarrow N(1-\mu) = 2\ ma$

$\Rightarrow 2(1-(P/10)) = 2\times 0.3\times 2$

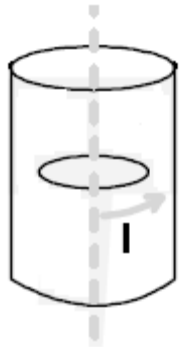
$\Rightarrow P = 4$

44. Four solid spheres each of diameter $\sqrt{5}$cm and mass 0.5 kg are placed with their centres at the corner of side 4cm. The moment of inertia of the system about the diagonal of the square is $N\times 10^{-4}\,kg-m^2$, then N is

44. **(9)** $I = 2\left[\left\{\frac{2}{5}mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right\} + \left\{\frac{2}{5}mr^2\right\}\right]$

$= 9\times 10^{-4}\,kg-m^2$

45. A long circular tube of length 10m and radius 0.3 m carries a current/along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0\cos(300t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300t)$, then 'N' is



45 **(6)** $I = iN \Rightarrow i = (I/M)$

[N: No. of turns per unit length assuming the tube as solenoid]

$B = \mu_0 ni = \mu(N/L)i = (\mu_0 I/L)$

$\therefore\ \phi = Ba = [(\mu_0 I/L).A]$

$\Rightarrow\ e = -(d\phi/dt) = [-(\mu_0 A/L).(dl/dt)]$

$\therefore\ \mu = i_1 A\ = [(\mu_0 A^2/LR)(dI/dt), \text{where } A = \pi r_i^2]$

$\Rightarrow \mu = [(A^2/LR)\times 300]\mu_0 I_0 \sin(300t)$

$\Rightarrow N = 0.6\pi^2 = 6$

46. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap fil of side 'a'. The surface tension of the soap film is γ . The system of charges and planar film asre in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where 'k' is a constant . then N is

46. **(3)** Consider the upper half of the setup

For equilibrium of the system

$$\Rightarrow 2.\gamma\ a = 2\left[\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{a^2} + \frac{1}{4\pi\varepsilon_0}.\frac{q^2}{\left(\sqrt{2}a\right)^2}.\frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow\ \gamma a^3 = q^2\left[(1/4\pi\varepsilon_0).\left(1+\left(1/2\sqrt{2}\right)\right)\right]$$

$\therefore\ a = \text{constant } (q^2/\gamma)^{1/3} \qquad \therefore N = 3$

Alternately,

$F_{elec} \propto a^{-2}$ while $F_{ST} \propto \gamma a \quad \therefore (q^2/\gamma) \propto a^3$

MATHEMATICS PAPER - I
PART-III- MATHEMATICS
SECTION-1 (TOTAL MARKS: 21)
(SINGLE CORRECT ANSWER TYPE)

This section contains 7 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

---

47. Let P = $\left\{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\right\}$ and Q = $\left\{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\right\}$ be two sets. Then

(a) $P \subset Q$ and $Q - P \neq \varnothing$ (b) $Q \not\subset P$ (c) $P \not\subset Q$ (d) $P = Q$

47. **(d)** $P : \sin\theta - \cos\theta = \sqrt{2}\cos\theta \Rightarrow \tan\theta = 1 + \sqrt{2}$

$Q : \sin\theta + \cos\theta = \sqrt{2}\sin\theta \Rightarrow \tan\theta = \dfrac{1}{\sqrt{2}-1} = \sqrt{2}+1$

$\therefore P = Q$

48. Let the straight line x = b divide the area enclosed by $y = (1-x)^2$, y = 0 and x = 0 into two parts $R_1(0 \le x \le b)$ and $R_2(0 \le x \le 1)$ such that $R_1 - R_2 = \dfrac{1}{4}$. Then b equals

(a) $\dfrac{3}{4}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{1}{4}$

48. **(b)** $R_1 - R_2 = \dfrac{1}{4} \Rightarrow \displaystyle\int_0^b (1-x^2)\,dx - \int_b^1 (1-x)^2\,dx = \dfrac{1}{4} \Rightarrow \dfrac{1}{3} - \dfrac{2(1-b)^3}{3} = \dfrac{1}{4}$

$\Rightarrow (1-b)^3 = \dfrac{1}{8} \Rightarrow b = \dfrac{1}{2}$

49. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \ge 1$, then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is

(a) 1 (b) 2 (c) 3 (d) 4

49. **(c)** Since $\alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha$

and $\beta^2 - 6\beta - 2 = 0 \Rightarrow \beta^2 - 2 = 6\beta$

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8 \cdot 6\alpha - \beta^8 \cdot 6\beta}{2(\alpha^9 - \beta^9)} = 3$$

50. A straight line L through the point (3, -2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x – axis, then the equation of L is

(a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

50. **(b)** If required line has slope = m

$$\Rightarrow \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \tan 60^o \Rightarrow m = 0, \sqrt{3}$$

Since, the line intersects the x – axis

$\Rightarrow$ equations is, $(y+2) = \sqrt{3}(x-3) \Rightarrow y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

51. Let (x_0, y_0) be the solution of the following equations

$(2x)^{\ln 2} = (3y)^{\ln 3}$

$3^{\ln x} = 2^{\ln y}$

Then x_0 is

(a) 1/6 (b) 1/ 3 (c) ½ (d) 6

51. **(c)** Since $(\ln 2)(\ln 2 + \ln x) = (\ln 3)(\ln 3 + \ln y)$ and lnx . ln 3 – ln y. ln 2 = 0

$\Rightarrow (\ln 2)\cdot(\ln x) - \ln y \cdot \ln 3 = (\ln 3)^2 - (\ln 2)^2$ …….(i)

and $\ln x.\ln 3 - \ln y.\ln 2 = 0$ ……(ii)

Using (i) × ln 2 – (ii) × ln 3, we get

$\ln x.\left\{(\ln 2)^2 - (\ln 3)^2\right\} = (\ln 2)\left\{(\ln 3)^2 - (\ln 2)^2\right\}$

$\Rightarrow \ln x = -\ln 2 \Rightarrow x = 1/2$

52. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is

(a) $(1/4)\ln(3/2)$ (b) $(1/2)\ln(3/2)$ (c) $\ln(3/2)$ (d) $(1/6)\ln(3/2)$

52. **(a)** $I = \int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$

$x^2 = t \quad \Rightarrow 2xdx = dt$

$$\Rightarrow I = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t}{\sin t + \sin(\ln 6 - t)} dt$$

Using $\int_a^b f(x)dx = \frac{1}{2}\int_a^b (f(x) + f(a+b-x))dx$

$$\Rightarrow I = \frac{1}{4}\int_{\ln 2}^{\ln 3}\left(\frac{\sin t}{\sin t + \sin(\ln 6 - t)} + \frac{\sin(\ln 6 - t)}{\sin(\ln 6 - t) + \sin t}\right)dt$$ [since ln 3 + ln 2 = ln 6]

$$\Rightarrow I = \frac{1}{4}\int_{\ln 2}^{\ln 3} dt = \frac{1}{4}[t]_{\ln 2}^{\ln 3} = \frac{1}{4}\ln\frac{3}{2}$$

53. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\bar{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $1/\sqrt{3}$ is given by

(a) $\hat{i} - 3\hat{j} + 3\hat{k}$ (b) $-3\hat{i} - 3\hat{j} - \hat{k}$ (c) $3\hat{i} - \hat{j} + 3\hat{k}$ (d) $\hat{i} + 3\hat{j} - 3\hat{k}$

53. **(c)** $\bar{v} = \lambda\left(\hat{i} + \hat{j} + \hat{k}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$

$$\frac{\vec{v}.\vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}} \Rightarrow (\lambda + \mu) - (\lambda - \mu) - (\lambda + \mu) = 1 \Rightarrow \mu = \lambda + 1$$

$$\bar{v} = \hat{i}(2\lambda + 1) - \hat{j} + \hat{k}(2\lambda + 1)$$

Only option (c) satisfies this one.

SECTION-II (TOTAL MARKS: 16)
(Multiple Correct Choice Type)

This section contains 4 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE OR MORE** may be correct.

54. Let f: R $\rightarrow$ R be a function such that $f(x+y) = f(x)+f(y),\ \forall x, y \in R$
If f(x) is differentiable at x = 0, then
(a) f(x) is differentiable only in a finite interval containing zero.
(b) f(x) is continuous $\forall x \in R$
(c) $f'(x)$ is constant $\forall x \in R$
(d) f(x) is differentiable except at finitely many points

54. **(b) (c)** $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h} = \lim_{h\to 0}\frac{f(h)-f(0)}{h}$
Put $x = y = 0,\ f(0) = 0$
$f'(x) = f'(0)$
$f(x) = f'(0)x + c$
$f(x) = kx + c,\ c = 0$ as $f(0) = 0$
$\therefore$ Function is continuous. f'(x) is constant.

55. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2+4y^2=4$.
If the hyperbola passes through a focus of the ellipse, then
(a) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$
(b) a focus of the hyperbola is (2, 0)
(c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$
(d) the equation of the hyperbola is $x^2-3y^2=3$

55. **(b) (d)** $\frac{x^2}{4}+\frac{y^2}{1}=1,\ a^2=4, b^2=1, e^2=1-(b^2/a^2) \Rightarrow e=\sqrt{3}/2$
eccentricity of hyperbola = $2/\sqrt{3}$
Focus of ellipse = $(\pm ae, 0) = (\pm\sqrt{3}, 0)$
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ passes through focus $(\pm\sqrt{3}, 0)$
$\therefore a^2 = 3 \qquad \therefore b^2 = a^2(e^2-1) = 1$
$\therefore$ Equation of hyperbola: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{3}-\frac{y^2}{1}=1$
$x^2-3y^2=3$
Focus of hyperbola $\equiv (\pm ae, 0) = (\pm 2, 0)$

56. Let M and N be two 3×3 non – singular skew – symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2\left(M^TN\right)^{-1}\left(MN^{-1}\right)T$ is equal to

(a) M^2 (b) $-N^2$ (c) $-M^2$ (d) MN

56. **(c)** Since $M^T=-M,\ N^T=-N$

$$M^2N^2\left(M^TN\right)^{-1}\left(MN^{-1}\right)^T$$
$$=M^2N^2\left(-MN\right)^{-1}\left(MN^{-1}\right)^T=M^2N^2\left(N^{-1}\cdot\left(-M\right)^{-1}\right)\cdot\left(N^T\right)^{-1}\cdot\left(-M\right)$$
$$=M^2N\left(-M\right)^{-1}\cdot\left(-N\right)^{-1}\cdot\left(-M\right)=M\left(MN\right)\left(MN\right)^{-1}\left(-M\right)=-M^2$$

57. The vector (s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is /are

(a) $\hat{j}-\hat{k}$ (b) $-\hat{i}+\hat{j}$ (c) $\hat{i}-\hat{j}$ (d) $-\hat{j}+\hat{k}$

57. **(a) (d)** Let $\vec{a}=\lambda\left(\hat{i}+\hat{j}+2\hat{k}\right)+\mu\left(\hat{i}+2\hat{j}+\hat{k}\right)$

$$=\left(\lambda+\mu\right)\hat{i}+\left(\lambda+2\mu\right)\hat{j}+\left(2\lambda+\mu\right)\hat{k}$$
$$\vec{a}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=0\quad\therefore\lambda=-\mu\quad\therefore\vec{a}=\lambda\left(-\hat{j}+\hat{k}\right)$$

Taking $\lambda=1,\quad\vec{a}=-\hat{j}+\hat{k}$

$\lambda=-1,\ \vec{a}=\hat{j}-\hat{k}$

SECTION-III (TOTAL MARKS: 15)
(Paragraph Type)

This section contains **2 paragraphs**. Based upon one of the paragraphs 2 multiple choice questions and based on other paragraph 3 multiple choice questions have to be answered. Each of these has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for question Nos 58 and 59

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2.Now 1 ball is drawn at random from U_2.

58. The probability of the drawn ball from U_2 being white is
(a) 13/30 (b) 23/30 (c) 19/ 30 (d) 11/30

58. **(b)**

$$P = \underbrace{\frac{1}{2}\left[\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right]}_{\text{For appearing head}} + \underbrace{\frac{1}{2}\left[\frac{{}^3C_1\times{}^2C_1}{{}^5C_2}\times\frac{2}{3}+\frac{{}^3C_2}{{}^5C_2}\times 1+\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}\right]}_{\text{For appearing tail}} = 23/30$$

59. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is
(a) 17/23 (b) 11/23 (c) 15/23 (d) 12/23

59. **(d)**

Using Bayes theorem

$$P = \frac{\frac{1}{2}\left[\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right]}{\frac{1}{2}\left[\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right]+\frac{1}{2}\left[\frac{{}^3C_1\times{}^2C_1}{{}^5C_2}\times\frac{2}{3}+\frac{{}^3C_2}{{}^5C_2}\times 1+\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}\right]} = 12/23$$

Paragraph for question Nos 60 and 62

Let a, b and c be three real numbers satisfying

$$\begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \qquad \ldots\ldots (E)$$

60. If the point P(a, b, c), with reference to (E), lies on the plane 2x + y + z = 1, then the value of 7a + b + c is
(a) 0 (b) 12 (c) 7 (d) 6

60. **(d)** $D = \begin{vmatrix} 1 & 8 & 7 \\ 9 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$\therefore$ system has non trivial solution.
If c = k, b = -6/7 k, a = -k/7
$\because$ 2a + b+ c = 1
$-(2k/7)-(6k/7)+k = 1 \Rightarrow k = -7$

$\therefore P(a,b,c) \equiv (1,6,-7)$

$7a + b + c = 7 + 6 - 7 = 6$

61. Let ω be a solution of $x^3 - 1 = 0$ with $\text{lm}(\omega) > 0$. If a = 2 with b and c satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to

(a) -2 (b) 2 (c) 3 (d) -3

61. **(a)** a =2, k = -14

c = -14, b = 12, a = 2

$$\frac{3}{\omega^2} + \frac{1}{\omega^{12}} + \frac{3}{\omega^{-14}} = \frac{3}{\omega^2} + \frac{1}{(\omega^3)^4} + \frac{3(\omega^3)^5}{\omega}$$

$$= \frac{3}{\omega^2} + 1 + \frac{3}{\omega} = \frac{3 + \omega^2 + 3\omega}{\omega^2} = \frac{3(1+\omega) + \omega^2}{\omega^2} = \frac{-3\omega^2 + \omega^2}{\omega^2} = -2$$

62. Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is

(a) 6 (b) 7 (c) 6/7 (d) ∞

62. **(b)** b = 6, k = -7, c = - 7, a = 1

Q.E. becomes $x^2 + 6x - 7 = 0$; a + b = -6 , ab = -7

$$\sum_{n=0}^{\infty}\left(\frac{\alpha+\beta}{\alpha\beta}\right)^n = \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n = 1 + \left(\frac{6}{7}\right) + \left(\frac{6}{7}\right)^2 + ... = \frac{1}{1 - \frac{6}{7}} = 7$$

SECTION-IV (TOTAL MARKS: 28)
(INTEGER ANSWER TYPE)

This section contains **7 questions**. The answer to each of the questions is a **single – digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

63. Let f: $[1,\infty) \to [(2,\infty)$ be a differentiable function such that f(1) = 2, if

$$6\int_1^x f(t)\,dt = 3xf(x) - x^3$$

for all $x \geq 1$, then the value of f(2) is

63. **(6)** Differentiating the given relation,

$6f(x) = 3f(x) + 3xf'(x) - 3x^2 \Rightarrow f'(x) - \frac{f(x)}{x} = x$, which is in linear form with I.F = $\frac{1}{x}$

Hence, $f(x) = x^2 + cx$ with c = 1. $\therefore f(x) = x^2 + x$. $\therefore f(2) = 6$

64. If z is any complex number satisfying $|z - 3 - 2i| \leq 2$, then the minimum value of $|2z - 6 + 5i|$ is

64. **(5)** Expression $2\left|Z - 3 + \frac{5}{2}i\right|$ which is equivalent to double the distance between complex number z and $(3, -5/2)$ which lies on one the diameter x = 3 of circle given. Hence minimum value of expression = $2 \times \frac{5}{2} = 5$

65. Let $a_1, a_2, ...a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \leq p \leq 100$. For any integer n with $1 \leq n \leq 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

65. **(9)**

$$\frac{S_m}{S_n} = \frac{a_1 + a_2 + ... + a_{5n}}{a_1 + a_2 + ... + a_n} = \frac{\frac{5n}{2}\{2.3 + (5n-1)d\}}{\frac{n}{2}\{2.3 + (n-1)d\}}$$

$$\lambda = \frac{5(6 + 5nd - d)}{(6 + nd - d)} \qquad \left[\lambda = \frac{S_m}{S_n}\right]$$

$\therefore \lambda$ independent of n when d = 6

$\therefore a_2 = a_1 + d = 9$

66. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is

66. **(2)** Extremities of L.R. are (2,4) & (2, -4) $\Rightarrow \Delta_1 = 6$
Points of intersection of tangents at these points are (-2, 0), (1, 3) & (-1, -1) $\Rightarrow \Delta_2 = 3$

67. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is .

67. **(1)** Put $k = \tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right) \Rightarrow \tan k = \frac{\sin\theta}{\sqrt{\cos 2\theta}}$

$$\Rightarrow \tan^2 k = \frac{\sin^2\theta}{1 - 2\sin^2\theta} \Rightarrow \sec^2 k - 1 = \frac{1}{\operatorname{cosec}^2\theta - 2}$$

$$\Rightarrow \sec^2 k = \frac{\operatorname{cosec}^2\theta - 1}{\operatorname{cosec}^2\theta - 2} \Rightarrow \sin^2 k = \frac{1}{\cot^2\theta} = \tan^2\theta$$

$$\Rightarrow \sin k = \tan\theta \quad \therefore f(\theta) = \sin k = \tan\theta \quad \text{Hence. } \frac{d(f(\theta))}{d(\tan\theta)} = 1$$

68. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with a > 0 is

68. **(8)** Using A.M $\geq$ G.M

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{8} \geq 1$$

$\therefore$ Minimum value of required expression = 8 (for a = 1).

69. The positive integer value of n > 3 satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

69. **(7)** $\frac{1}{\sin x} = \frac{1}{\sin 2x} + \frac{1}{\sin 3x}$ { where $x = (\pi / n)$ }

or $\frac{\sin 3x - \sin x}{\sin x . \sin 3x} = \frac{1}{\sin 2x}$ or $\frac{2\sin x . \cos 2x}{\sin x . \sin 3x} = \frac{1}{\sin 2x}$

sin4x = sin 3x [$\because \sin x \neq 0$]

$\therefore$ 4x = 3x (not possible $\because$ $x \neq 0$)

or $4x = \pi - 3x \Rightarrow 7x = \pi \Rightarrow 7.(\pi / n) = \pi \quad \therefore n = 7$