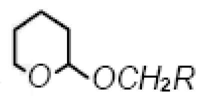
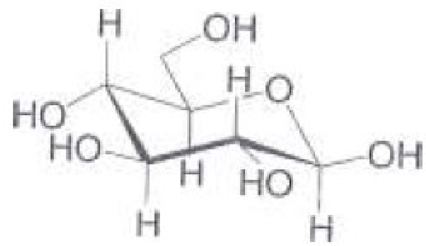
CHEMISTRY PAPER II
PART-1- CHEMISTRY
SECTION-1 (TOTAL MARKS: 24)
(SINGLE CORRECT CHOICE TYPE)

This section contains 8 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. The major product of the following reaction is
(a) A hemiacetal (b) An acetal (c) an ether (d) an ester

1. **(b)**  product is an acetal

2. The following carbohydrate is



(a) A ketohexose (b) An aldohexose
(c) An $\alpha-$ furanose d) An $\alpha-$ Pyranose

2. **(b)**

3. Oxidation states of the metal in the minerals haematite and magnetite, respectively, are
(a) I,III in haematite and III in magnetite
(b) II,III in haematite and II in magnetite
(c) II in haematite and II, III in magnetite
(d) III in haematite and II, III in magnetite

3. **(d)** Hamatite is Fe_2O_3 where iron is in III oxidation state.
Magentite is Fe_3O_4 which is a mixed oxide where iron exhibits both II and III oxidation state.

4. Among the following complexes (K-P)
$K_3\left[Fe(CN)_6\right](K), \left[CO(NH_3)_6\right]Cl_3(L), Na_3\left[Co(oxalate)_3\right](M), Ni(H_2O)_6]Cl_2(N), K_2\left[Pt(CN)_4\right](O)$
and $\left[Zn(H_2O)_6\right](NO_3)_2(P)$
the diamagnetic complexes are
(a) K,L,M,N (b) K,M,O,P (c) L,M,O,P (d) L,M,N,O

4. **(c)** K has one unpaired ‘e’
N all octahedral complexes are outer orbital complex

5. Passing H_2S gas into a mixture of $Mn^{2+}, Ni^{2+}, Cu^{2+}$ and Hg^{2+} ions in an acidified aqueous solution precipitates

(a) CuS and HgS (b) MnS and CuS

(c) MnS and NiS (d) NiS and HgS

5. **(a)** CuS and HgS get precipitated

6. Consider the following cell reaction:

$2Fe_{(s)} + O_{2(g)} + 4H^+_{(aq)} \rightarrow 2Fe^{2+}_{(aq)} + 2H_2O(\ell)$ $E^o = 1.67V$

At $\left[Fe^{2+}\right] = 10^{-3} M, P(O_2) = 0.1$ atm and pH = 3, the cell potential at 25^oC is

(a) 1.47 V (b) 1.77 V (c) 1.87 V (d) 1.57 V

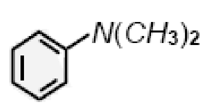
6. **(d)** $1.67 - \frac{.06}{(2)^2}\log\left(Fe^{+2}\right)^2 + \frac{.06}{4}\log\frac{(O_2)\left[H^+\right]^4}{H_2O} = 1.55V$

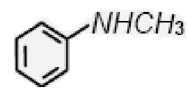
7. The freezing point (in ^oC) of a solution containing 0.1g of $K_2\left[Fe(CN)_6\right]$(Mol.Wt.329) in 100g of water $\left(K_f = 1.86 K\,kg\,mol^{-1}\right)$ is

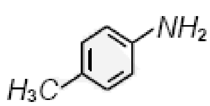
(a) -2.3×10^{-2} (b) -5.7×10^{-2} (c) -5.7×10^{-3} (d) -1.2×10^{-2}

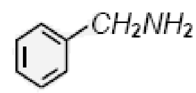
7. **(a)** $\Delta T_f = (1.86\times0.1\times20\times4)/329 = 2.3\times10^{-2}$

8. Amongst the compound given, the one that would form a brilliant colored dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of $\beta-$naphthol is

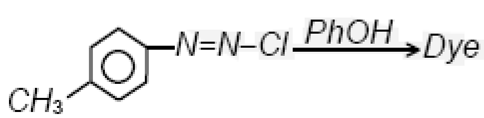
(a) 

(b) 

(c) 

(d) 

8. **(c)**



SECTION –II (TOTAL MARKS: 16)
(MULTIPLE CORRECT ANSWER(S) TYPE)

This section contains **4 multiple choice questions**. Each question has four choices (a),(b),(c) and (d) out of which **ONE OR MORE** may be correct.

9. For the first reaction

$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$

(a) The concentration of the reactant decreases exponentially with time.
(b) The half-life of the reaction decreases with increasing temperature.
(c) The half-life of the reaction depends on the initial concentration of the reactant.
(d) The reaction proceeds to 99.6% completion in eight half-half duration.

9. **(abc)** $\frac{-693}{t_{1/2}} = \frac{2.3}{8 \times t_{1/2}} \log \frac{100}{[A]_t}$

$2.4 = \log \frac{100}{[A]_t}$

$\therefore$ $[A]_t = 100/250 = 0.4$

$\therefore$ 99.6%

10. The correct functional group X and the reagent / reaction conditions Y in the following scheme are

(a) $X = COOCH_3, Y = H_2 / Ni / heat$
(b) $X = CONH_2, Y = H_2 / Ni / heat$
(c) $X = CONH_2, Y = Br_2 / NaOH$
(d) $X = CN, Y = H_2 / Ni / heat$

10. **(abcd)** Alcohol gives polyster
NH_2 group gives polyamide

11. The equilibrium

$2Cu^{I} \Leftrightarrow Cu^{o} + Cu^{II}$

In aqueous medium at $25^{o}C$ shifts towards the left in the presence of

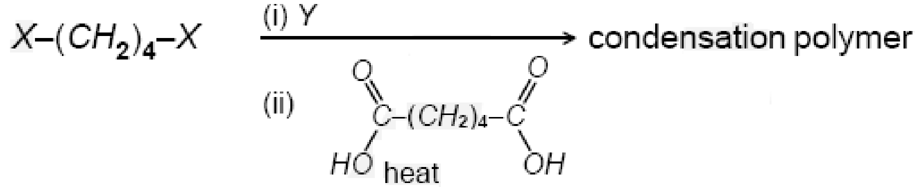
(a) NO_3^- (b) Cl^- (c) SCN^- (d) CN^-

11. **(bcd)** Cl^-, SCN^- and CN^- form insoluble compound of $CuCl, CuSCN$ and $CuCN$ which drags the equilibrium reaction in backward direction.

12. Reduction of the metal centre in aqueous permanganate ion involves

(a) 3 electrons in the neutral medium
(b) 5 electrons in neutral medium
(c) 3 electrons in alkaline medium
(d) 5 electrons in acidic medium

12. **(ad)**



SECTION-III (TOTAL MARKS: 24)
(INTEGER ANSWER TYPE)

This section contains **6 questions.** The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

13. Among the following, the number of compounds that can react with PCl_5 to give $POCl_3$ is
$O_2.CO_2, SO_2, H_2O, H_2SO_4, P_4O_{10}$

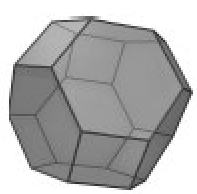
13. **(4)**
$$PCl_5 + H_2O \rightarrow POCl_3 + 2HCl$$
$$H_2SO_4 + PCl_2 \rightarrow 2POCl_3 + SO_2Cl_2 + 2HCl$$
$$SO_2 + PCl_2 \rightarrow POCl_3 + SOCl_2$$
$$P_4O_{10} + 6PCl_5 \rightarrow 10POCl_3$$

14. In 1 L saturated solution of $AgCl\left[K_{sp}\left(AgCl\right) = 1.6\times10^{-10}\right]$, 0.1mol of CuCl $\left[K_{sp}\left(CuCl\right) = 1.0\times10^{-6}\right]$ is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x}. The value of "x" is

14. **(7)**
$$\left[Cl^-\right] = \left[Ag^+\right] + \left[Cu^+\right]$$
$$\left[Cl^-\right]^2 = 1.6\times10^{-10} + 10^{-6} \approx 10^{-6}$$
$$\therefore \left[Cl^-\right] = 10^{-3}$$
$$\therefore \left[Ag^+\right] = 1.6\times10^{-7} \quad \therefore x = 7$$

15. The number of hexagonal faces that are present in truncated octahedron is

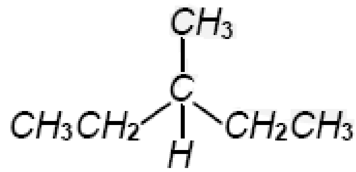
15. **(8)**



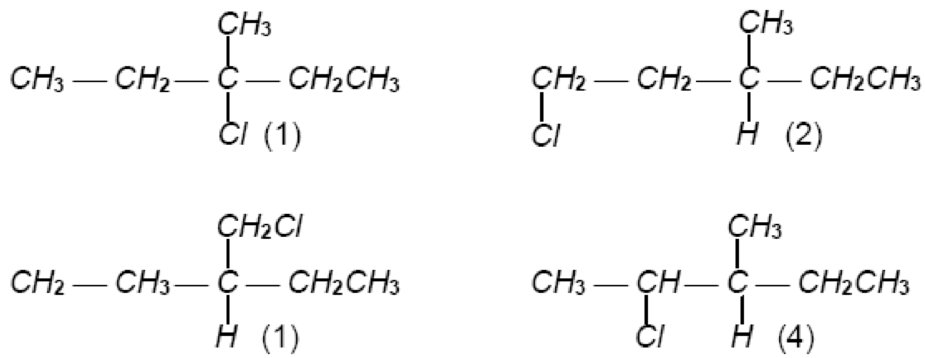
16. The total number of contributing structures showing hyperconjugation (involving C-H bonds) for the following carbocation is

16. **(6)** There are six $\alpha -$ hydrogens.

17. The maximum number of isomers (including stereoisomers) that are possible on monochlorination of the following compound, is



17. **(8)**



Maximum number of isomers = 8

18. The volume (in mL) of 0.1M $AgNO_3$ required for complete precipitation of chloride ions present in 30mL of 0.01 M solution of $\left[Cr\left(H_2O\right)_5 Cl\right]Cl_2$ as silver chloride is close to

18. **(6)** $30 \times 0.01 \times 2 = 0.1 \times V$

$\therefore V = 6$ ml

SECTION –IV (TOTAL MARKS: 16)
(MATRIX-MATCH TYPE)

This section contains **2 questions**. Each question has **four statements** (a, b, c and d) given in **Column I** and five statements (p, q, r s and t) in **Column II**. Any given statement in **Column I** can have correct matching with **ONE** or **MORE** statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

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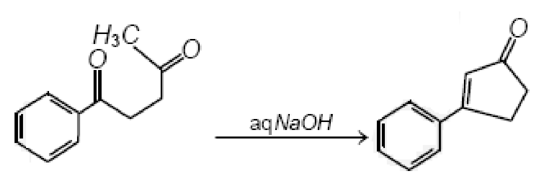
19. Match the transformations in Column I with appropriate options in Column II

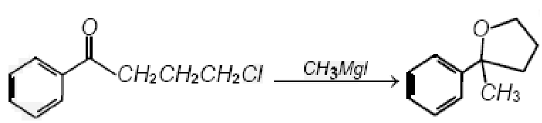
Column I		Column II	
(a)	$CO_2(s) \rightarrow CO_2(g)$	(p)	Phase transition
(b)	$CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$	(q)	Allotropic change
(c)	$2H_{\bullet} \rightarrow H_2(g)$	(r)	ΔH is positive
(d)	$P_{(white,solid)} \square\square\square\square \rightarrow P_{(red,solid)}$	(s)	ΔS is positive
		(t)	ΔS is negetive

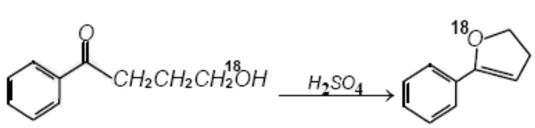
19. $a \rightarrow (p,r,s)$ $b \rightarrow (r,s)$ $c \rightarrow (t)$ $d \rightarrow (q,r)$

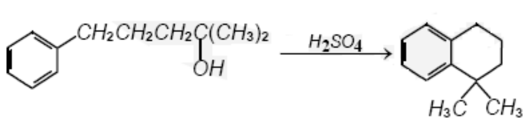
20. Match the reactions in Column I with appropriate types of steps/ reactive intermediate involved in these reactions as given in Column II.

Column- I

(a)


(b)


(c)


(d)


Column- II

(p) Nucleopphilic substitution

(q) Electrophilic substitution

(r) Dehydration

(s) Nucleophilic addition

(t) Carbanion

20. $a \rightarrow (r,s,t)$ $b \rightarrow (s,p)$ $c \rightarrow (s,r)$ $d(q,r)$

PHYSICS PAPER-II
PART II-PHYSICS
SECTION –I (TOTAL MARKS: 24)
(SINGLE CORRECT CHOICE TYPE)

This Section contains **8 multiple choice questions**. Each question ha four choice (a),(b),(c) and (d) out of which **ONLY ONE** is correct.

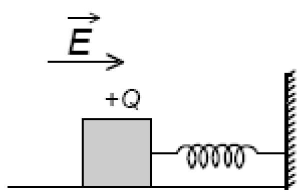
21. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5mm and there are 50divisions on the circular sacle. The reading on the main scale is 2.5mm and that on the circular scale is 20 divisions. If the measured mass of the ball has relative error of 2%, the relative percentage error in the density is
(a) 0.9% (b) 2.4% (c) 3.1% (d) 4.2%

21. **(c)** LC = (0.5/50)=10^{-2}mm
Reading = 2.5 + 20$\times$ 10^{-2} = 2.7 mm

$$\rho = \frac{m}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \Rightarrow (\Delta\rho/\rho) = \left[(\Delta m/m) + 3.(\Delta d/d)\right]$$

$$= \left[2 + 3\times\left(10^{-2}/2.7\right)\times 100\right] = 3.1\%$$

22. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge +Q on its surface. If now a uniform electric filed $\vec{E}$ is switched-on as shown, the SHM of the block will be:

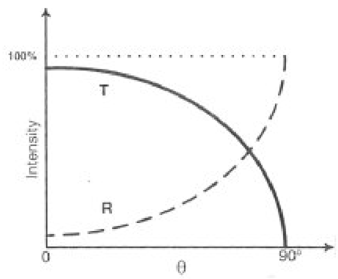


(a) of the same frequency and with shifted mean position
(b) of the same frequency and with the same mean position
(c) of changed frequency and with shifted mean position
(d) of changed frequency and with the same mean position.

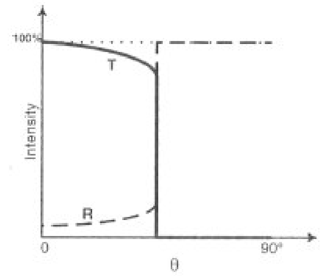
22. **(a)** Restoring force remains same, so frequency does not change.
Only mean position shifted rightward by (EQ/K)

23. A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

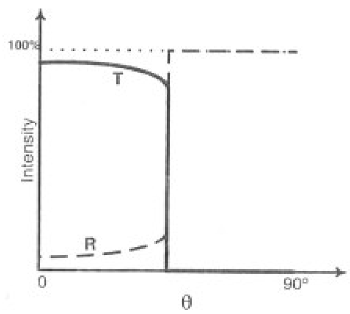
(a)



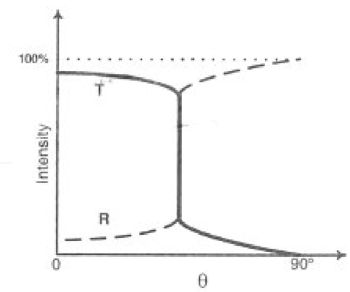
(b)



(c)



(d)



23. **(c)** for $0 < \theta < \theta_C$ $\quad T > R$

& $\theta \geq \theta_C$ $\quad T = 0, R = 100\%$

Also $\theta \to 0$ $\quad T < 100\%, R > 0\%$

24. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

(a) $(1/2)mV^2$ (b) mV^2 (c) $(3/2)mV^2$ (d) $2mV^2$

24. **(b)** $\left(mV^2/r\right) = \left(GmM_e/r^2\right) \Rightarrow r = \left(GM_e/V^2\right)$

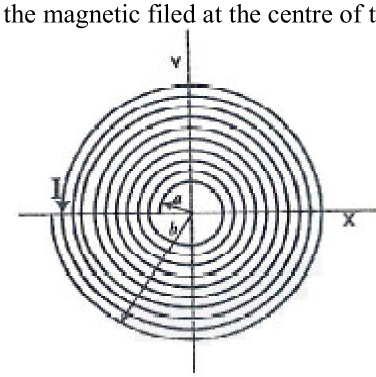
Now using energy conservation for particle

$\left[(1/2)Mu^2 - \left(GM_e m/r\right)\right] = 0$

$$\Rightarrow \frac{1}{2}mu^2 - m\frac{GM_e}{\left(\frac{GM_e}{V^2}\right)} = 0$$

$\Rightarrow (1/2)Mu^2 = mV^2 = K.E$

25. A long insulated copped wire is closely wound as a spiral of 'N turns. The spiral has inner radius 'a' and out radius 'b'. The spiral lies in the X-Y plane and a steady current 'l' flows through the wire. The Z-component of the magnetic filed at the centre of the spiral is:



(a) $\frac{\mu_0 Nl}{2(b-a)}\ln\left(\frac{b}{a}\right)$ (b) $\frac{\mu_0 Nl}{2(b-a)}\ln\left(\frac{b+a}{b-a}\right)$

(c) $\frac{\mu_0 Nl}{2b}\ln\left(\frac{b}{a}\right)$ (d) $\frac{\mu_0 Nl}{2b}\ln\left(\frac{b+a}{b-a}\right)$

25. **(a)** No. of turns per unit length = $\left[Nl(b-a)\right]$

No. of turns in thickness dx = $\left[Nl(b-a)\right]dx$

Magnetic field B = $\int dB = \int_a^b \frac{\mu_0 Nl}{2(b-a)x}dx$

$= \left[\mu_0 Nl/2(b-a)\right]\ln(b/a)$

26. A point mass is subjected to two simultaneous sinusoidal displacement in x-direction, $x_1(t) = A\ \sin\omega t$ and $x_2(t) = A\sin\left[\omega t + (2\pi/3)\right]$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

(a) $\sqrt{2}\,A, (3\pi/4)$ (b) $A, (4\pi/3)$

(c) $\sqrt{3}\ A, (5\pi/6)$ (d) $A, (\pi/3)$

26. **(b)** $(x_1 + x_2) = A\sin\omega t + A\sin(\omega t + 2\pi/3)$

$= A\left[(1/2)\sin\omega t + \left(\sqrt{3}/2\right)\cos\omega t\right]$ $= A\sin\left(\omega t + (\pi/3)\right)$

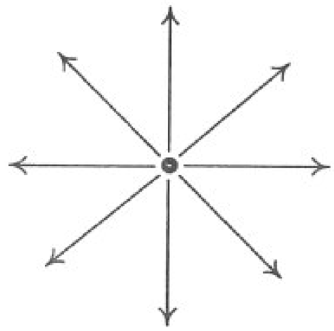
Now $x_1 + x_2 + x_3 = 0$

$\Rightarrow\ A\sin\left[\omega t + (\pi/3)\right] = -B\sin(\omega t + \phi)$

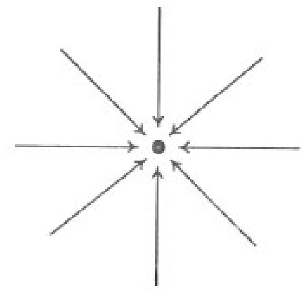
$\Rightarrow\ B = A \quad \& \quad \phi = 4(\pi/3)$

27. Which of the filed patterns given below is valid for electric field as well as for magnetic field?

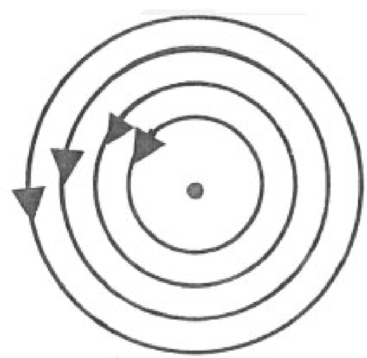
(a)



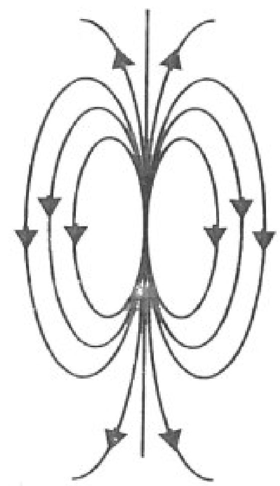
(b)



(c)

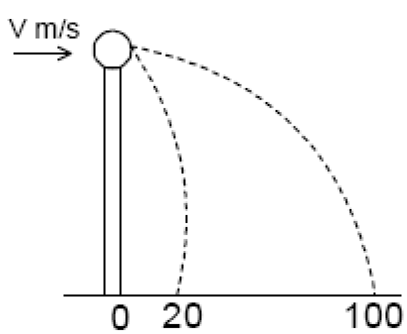


(d)



27. **(c)** Electric field produced by the time varying magnetic field is circular.
For a long straight wire, the magnetic field lines are circles with their centres on the wire.

28. A ball of mass 0.2kg rests on a vertical post of height 5m. A bullet of mass 0.01kg, travelling with a velocity Vms^{-1} in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet ay a distance of 100m from the foot of the post. The initial velocity V of the bullet is :



(a) $250ms^{-1}$ (b) $250\sqrt{2}\ ms^{-1}$ (c) $400ms^{-1}$ (d) $500ms^{-1}$

28. **(d)** Time of flight after collision for both are

$t = \sqrt{(2h/g)} = 1\text{sec}$

$\therefore\ V_{bullet} = (100/1) = 100ms^{-1}$

$V_{ball} = (20/1) = 20ms - 1$

From conservation of Linear momentum

$m_{bullet}u_{bullet} = [(m_{bullet} \times v_{bullet}) + (m_{ball} \times v_{ball})]$

$\Rightarrow\ u_{bullet} = v = [(0.01 \times 100) + (0.2 \times 20)]/0.01 \quad = 500ms^{-1}$

SECTION-II (TOTAL MAKS: 16)
(MULTIPLE CORRECT ANSWER TYPE)

This section contains 4 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE** or **MORE** many be correct.

29. Which of the following statement(s) is/are correct?

(a) If the electric filed due to a point charge varies as I^{-25} instead of I^{-2}, then the Gauss law will still be valid

(b) The Gauss law can be used to calculate the field distribution around an electric dipole

(c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$

29. **(cd)** For point charge, flux for a sphere if point charge is placed at the centre

$$= \frac{1}{4\pi\varepsilon_0 \left(r^{-2.5}\right)} \times 4\pi r^2 \qquad \text{[According to question]}$$

$$\neq \left(q / \varepsilon_0\right)$$

So Gauss law is not valid.

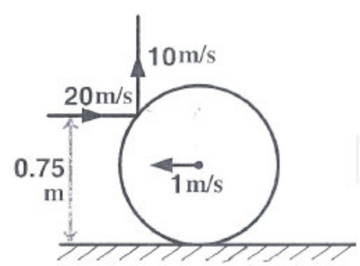
Electric field due to a dipole is not radially symmetric.

So Gauss law application does not give field distribution.

For electric field to be zero, directions of electric field must be opposite and equal in magnitude. This occurs only if sign of both the charges is same.

$$W_{A\to B}^{ext.agent} = -\frac{W_{elc.field}}{q(=1)} = \frac{U_B - U_A}{q(=1)} = V_B - V_A$$

30. A thin ring of mass 2kg and radius 0.5m is rolling without slipping on a horizontal plane with velocity $1ms^{-1}$. A small ball of mass 0.1kg moving with velocity $20ms^{-1}$ in the opposite direction, hits the ring at a height of 0.75m and goes vertically up with velocity $10ms^{-1}$. Immediately after the collision.



(a) the ring has pure rotation about its stationary CM

(b) the ring comes to a complete stop

(c) friction between the ring and the ground is to the left

(d) there is no friction between the ring and the ground

30. **(ac)** Applying CLM horizontal direction: $\left[(0.1\times 20)-(2\times 1)\right]=2V_{\text{CoM ring}}$

$\Rightarrow V_{\text{CoM}}=0$

Using CAM about contact point:

$$L_{\text{particle initially}}\ (\circlearrowright)+L_{\text{ring initially}}\ (\circlearrowleft)$$

$$=L_{\text{ring finally}}+L_{\text{particle finally}}\ (\circlearrowright)$$

From given data, initial angular momentum is anticlockwise.

$\therefore L_{\text{ring finally}}$ is anticlockwise

Hence friction on the ring acts leftward to oppose rotation.

31. A series R-C circuit is connected to AC voltage source. Consider two cases; (a) when C is without a dielectric medium and b when C is filled with dielectric of constant 4. The current I_R of the following is/are true ?

(a) $I^A{}_R > I^B{}_R$ (b) $I^A{}_R < I^B{}_R$ (c) $V^A{}_C > V^B{}_C$ (d) $V^A{}_C < V^B{}_C$

31. **(bc)** Z for 1st case, $Z_A=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$

Z for 2nd case, $Z_B=\sqrt{R^2+\frac{1}{(k\omega c)^2}}$

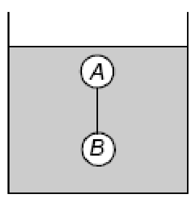
So $Z_B<Z_A$ So, $I^B{}_R > I^A{}_R$

Since, V_{source} is same

$V_{\text{Resistance}}$ increases for B

$V_{\text{Capacitor}}$ decreases for B

32. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



(a) $d_A<d_F$ (b) $d_B>d_F$ (c) $d_A>d_F$ (d) $d_A+d_B=2d_F$

32. **(abd)** Considering the F.B.D

$T=Vg(d_F-d_A)$ and $T=Vg(d_B-d_F)$

For both conditions, T > 0

$\Rightarrow d_F>d_A$ and $d_B>d_F$

Equating tension, $2d_F=d_B+d_A$

SECTION-III (TOTAL MARKS 24)
(INTEGER ANSWER TYPE)

This section contains **6 questions**. The answer to each of the question is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

33. A train is moving along a straight line with a constant acceleration 'a'. A body standing in the train throws a ball forward with a speed of $10ms^{-1}$, at an angle of 60^o to the horizontal. The boy has to move forward by 1.15m inside the train to catch the ball back at the initial height. The acceleration of the train, in ms^{-2} is

33. **(5)** Let u be the speed of the train at the time when by throws the ball.

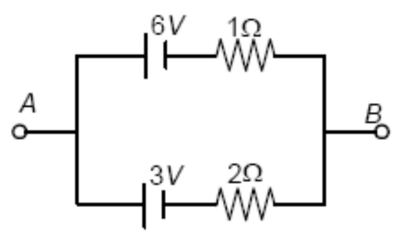
Time of flight $t_f = \left[\left(2\times 10\sin 60^o\right)/g\right] = \sqrt{3}\,\text{sec}$

$S_{train} = ut_f + \left(1/2\right)at_f^2$

$S_{ball} = ut_f + 10\cos 60^o t_f \qquad S_{ball} - S_{train} = 1.15$

$\Rightarrow \sqrt{3}u + 5\sqrt{3} - \sqrt{3}u - \left(3/2\right)a = 1.15 \qquad \Rightarrow a = 5\,ms^{-2}$

34. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is



34. **(5)** If V is p.d. across A & B , i_1 is current through 1Ω & i_2 is current through 2Ω , then

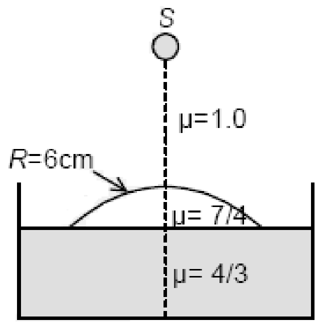
$V = 6 + i_1 = 3 + 2i_2$ ..(i)

$\Rightarrow i_1 = V - 6$ & $i_2 = \left[\left(V-3\right)/2\right]$

Also $i_1 + i_2 = 0$ ..(ii)

$\therefore V = 5V$

35. Water (with refractive index = (4/3)) in a tank is 18 cm deep. Oil of refractive index (7/4) lies on water making a convex surface of radius of curvature 'R = 6cm' as shown. Consider oil to act as thin lens. An object 'S' is place 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is



35. **(2)** Refraction at oil surface $(1/u_1)+(\mu_1/v_1)=[(\mu_1-1)/R]$

$\Rightarrow (1/24)+[(7/4)/v_1]=[((7/4)-1)/6] \Rightarrow v_1 = 21\text{cm}$

Refraction at water surface: $u_2 = -21\text{cm}$

$\therefore (1/-21)+[(4/3)/(7/4)](1/v_2)=0$

$\Rightarrow v_2 = 16$ cm (from water surface) $\therefore x = 18-16 = 2\text{cm}$

36. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free – space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^{z}$ (where 1 < A< 10). The value of 'Z' is

36. **(7)** $(KE)_{max} = E_{incident} - W = (12.42/2)\text{eV} - 4.7\text{eV} = 1.51\text{eV}$

& Maximum potential acquired (when maximum no. of photoelectrons emitted)

$V_{max} = [K(ne)/r]$ S / unit $\& (KE)_{max} = eV_{max}$

$\Rightarrow n = (15/14.4)\times 10^{7}$

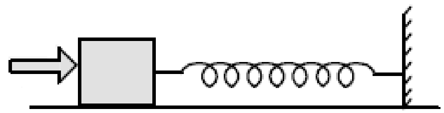
37. A series R – C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian s^{-1}. If the impedance of the R – C circuit is R $\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

37. **(4)** $Z^2 = R^2 \times X_C^2 \quad \Rightarrow R^2(1.25) = R^2 + [1/(\omega C)^2]$

$\Rightarrow R^2/4 = [1/(500C)^2] \quad \Rightarrow (RC)^2 = [4/(500)^2]$

$\Rightarrow RC = (1/250)\text{s} = 4$ ms

38. A block of mass 0.18 kg is attached to a spring of force – constant 2 Nm^{-1}. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un – stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in ms^{-1} is V = N/10. Then N is



38. **(4)** $(1/2)mV^2 = (1/2)Kx^2 + fx$

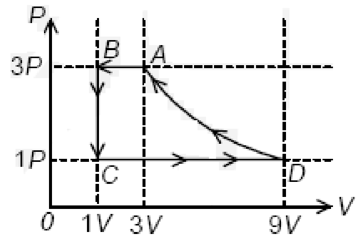
$\Rightarrow (1/2)\times(18\times10^{-2})V^2 = (1/2)\times2\times(6\times10^{-2})^2 + (18\times10^{-2})\times10\times10^{-1})\times(6\times10^{-2})$

$\Rightarrow V^2 = (144/900) \Rightarrow (N/10) = (12/30) \Rightarrow N = 4$

SECTION – IV (TOTAL MARKS: 16)
(MATRIX – MATCH TYPE)

This section contains **2 questions**. Each question **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

39. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P –V diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I.**



	Column I		**Column II**
(a)	Process $A \rightarrow B$	(p)	Internal energy decreases
(b)	Process $B \rightarrow C$	(q)	Internal energy increases
(c)	Process $C \rightarrow D$	(r)	Heat is lost.
(d)	Process $D \rightarrow A$	(s)	Heated is gained
		(t)	Work is done on the gas.

39. (a) $\rightarrow$ (p, r, t); (b) $\rightarrow$ (p, r); (c) $\rightarrow$ (q, s); (d) $\rightarrow$ (r, t)

$A \rightarrow B: \rightarrow$ IBC $\Rightarrow$ T $\downarrow$ as well as V($\downarrow$)

$\Rightarrow$ internal energy ($\downarrow$) as well as work is done on the gas.

also, $\Delta Q = nC_P. \ \Delta T$: (-ve) $\Rightarrow$ (p, r, t) is correct.

$B \rightarrow C : \rightarrow ICC \Rightarrow \Delta W = 0, \Delta T : -ve \quad \Rightarrow$ internal energy decreases.

also, $\Delta Q = nC_V \cdot \Delta T : (-ve) \quad \Rightarrow$ (p,r) is correct.

$C \rightarrow D : \quad \rightarrow IBE, \quad V(\uparrow)$

$\Rightarrow$ work done by the gas. also T ($\uparrow$) $\Rightarrow$ internal energy increases.

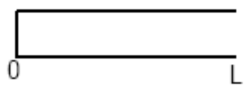
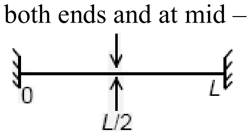
also, $\Delta Q = nC_P \Delta T : (+ve) \Rightarrow (q, s)$ is correct.

$D \rightarrow A : \rightarrow T_A = T_D \Rightarrow \Delta T = 0 \quad \Rightarrow \Delta U = 0 \ \ V(\downarrow) =$ work done on the gas.

We have, $\Delta Q = \Delta W + \Delta U; \qquad \Rightarrow \Delta Q = (-ve) + 0 = (-ve)$

$\Rightarrow (r, t)$ is correct

40. **Column I** shows four system, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_1. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves.

	Column I		Column II
(A)	Pipe closed at one end 	(p)	Longitudinal waves
(B)	Pipe open at both ends 	(q)	Transverse waves
(C)	Stretched wire clamped at both ends 	(r)	$\lambda_f = L$
(D)	Stretched wire clamped at both ends and at mid – point 	(s)	$\lambda_f = 2L$
		(t)	$\lambda_f = 4L$

40. A $\rightarrow$ (p,t), B $\rightarrow$ (p,s), C $\rightarrow$ (q,s), D $\rightarrow$ (q,r)

A $\rightarrow$ Longitudinal standing wave also, $(\lambda_f / 4) = L \Rightarrow \lambda_f = 4L \Rightarrow (p, t)$ is correct

B $\rightarrow$ Longitudinal standing wave also, $(\lambda_f / 2) = L \Rightarrow \lambda_f = 2L \Rightarrow (p, s)$ is correct

C $\rightarrow$ Longitudinal standing wave also, $(\lambda_f / 2) = L \Rightarrow \lambda_f = 2L \Rightarrow (q, s)$ is correct

D $\rightarrow$ Longitudinal standing wave also, $(\lambda_f / 2) + (\lambda_f / 2) = L \Rightarrow \lambda_f = L \Rightarrow (q, r)$

MATHEMATICS PAPER – II
PART III – MATHEMATICS
SECTION – I (TOTAL MARKS: 24)
(SINGLE CORRECT ANSWER TYPE)

This section contains 8 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

41. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then the set of all x satisfying (f o g o g o g) (x) = (g o g o f) (x), where (f o g) (x) = f (g (x)) , is

(a) $\pm\sqrt{n\pi}$, $n \in \{0,1,2,.......\}$

(b) $\pm\sqrt{n\pi}$, $n \in \{1,2,........\}$

(c) $(\pi/2) + 2n\pi$, $n \in \{......,-2,-1,0,1,2........\}$

(d) $2n\pi$, $n \in \{....,-2,-1,0,1,2......\}$

41. **(a)** (f o g o g o f) (x) = (g o g o f) (x)

$\Rightarrow \left[\sin\sin x^2\right]^2 = \sin\sin x^2 \Rightarrow \sin\sin x^2 = 0$ or 1

$\Rightarrow x = \pm\sqrt{n\pi},\ n \in \{0,1,2......\}$

42. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then the locus of P is

(a) $x^2 = y$ (b) $y^2 = 2x$ (c) $y^2 = x$ (d) $x^2 = 2y$

42. **(c)** $y = 2\sqrt{x}$

Let P be (h, k) $x = 4h$, $\sqrt{x} = 2k \Rightarrow k^2 = h$

$\therefore$ Locus of P is $y^2 = x$.

43. Let P (6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x – axis at (9, 0), then the eccentricity of the hyperbola is

(a) $\sqrt{(5/2)}$ (b) $\sqrt{(3/2)}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

43. **(b)** Normal to hyperbola at P (6, 3)

$$\frac{x-6}{6/a^2} = \frac{y-3}{-3/b^2}$$

$y = 0 \Rightarrow x = 6 + 6\left(b^2/a^2\right) = 9 \Rightarrow b^2/a^2 = 1/2$

$e^2 = 1 + \left(b^2/a^2\right) = 3/2 \Rightarrow e = \sqrt{(3/2)}$

44. A value of b for which the equations
$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$
have one root in common is
(a) - $\sqrt{2}$ (b) - i$\sqrt{3}$ (c) i $\sqrt{5}$ (d) $\sqrt{2}$

45. **(b)** α be the common root
$\alpha^2 + b\alpha - 1 = 0$
$\alpha^2 + \alpha + b = 0$
$\Rightarrow \alpha^2 = \frac{b^2+1}{1-b}, \ \alpha = \frac{1+b}{b-1} \ \therefore \ b^3 + 3b = 0 \Rightarrow b^2 = -3 \ \therefore b = -i\sqrt{3}$

45. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non – singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is
(a) 2 (b) 6 (c) 4 (d) 8

45. **(a)** $\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$
$\Rightarrow 1 - (a+c)\omega + ac\omega^2 \neq 0 \ \therefore \ a + c \neq -1 \quad ac \neq 1$
Clearly a = ω or ω^2 or c = ω or ω^2
but $c \neq \omega^2$ as it makes the determinant zero.
also $a \neq \omega^2$
$\therefore a = \omega, b = \omega^2$ or $\omega, c = \omega$
$\therefore$ No of such matrices = 2

46. The circle passing through the point (-1, 0) and touching the y – axis at (0, 2) also passes through the point
(a) (-3/2, 0) (b) (-5/2,2) (c) (-3/2, 5/2) (d) (-4, 0)

46. **(d)** Let C be (h, 2)
Equation of circle:
$\because$ circle passes through (-1, 0) , h = -5/2
Equation of circle will be $(x + (5/2))^2 + (y - 2)^2 = 25/4$
Only (-4, 0) satisfies

47. If $\lim_{x\to 0}\left[1 + x\ln\left(1+b^2\right)\right]^{1/x} = 2b\sin^2\theta$, b > 0 and $\theta \in (-\pi, \pi]$ then the value of θ is
(a) $\pm\pi/4$ (b) $\pm\pi/3$ (c) $\pm\pi/6$ (d) $\pm\pi/2$

47. **(d)** $1 + b^2 = 2b\sin^2\theta \Rightarrow b^2 - 2b\sin^2\theta + 1 = 0$
$\Rightarrow b = \frac{2\sin^2\theta \pm \sqrt{4\sin^4\theta - 4}}{2} = \sin^2\theta \pm \sqrt{-\cos^2\theta\left(1+\sin^2\theta\right)}$
For b to be real, cos $\theta = 0 \ \Rightarrow \theta = \pm\pi/2$

48. Let f: $\left[-1,2\right]\to\left(0,\infty\right]$ be a continuous function such that f(x) = f(1 –x) for all x $\in\left[-1,2\right]$. Let $R_1=\int_{-1}^{2}xf\left(x\right)dx$, and R_2 be the area of the region bounded by y = f(x), x = -1, x = 2, and the x – axis. Then

(a) $R_1=2R_2$ (b) $R_1=3R_2$ (c) $2R_1=R_2$ (d) $3R_1=R_2$

48. **(c)** $R_1=\int_{-1}^{2}\left(1-x\right)f\left(1-x\right)dx=R_2-R_1\Rightarrow 2R_1=R_2$

SECTION – II (TOTAL MARKS: 16)
MULTIPLE CORRECT CHOICE TYPE

This section contains 4 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE** may be correct.

49. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), the L is given by
(a) $y - x + 3 = 0$ (b) $y + 3x - 33 = 0$
(c) $y + x - 15$ (d) $y - 2x + 12 = 0$

49. **(a,b,d)** Equation of normal $y = mx - 2am - am^3$ i.e $y = mx - 2m - m^3$
Passing through (9, 6) $\therefore m^3 - 7m + 6 = 0 \Rightarrow m = -3, 2, 1$
$\therefore$ Equation of normals are $y + 3x - 33 = 0$, $y - 2x + 12 = 0$, $y - x + 3 = 0$

50. Let f: $(0,1) \to R$ be defined by $f(x) = \dfrac{b-x}{1-bx}$, where b is a constant such that $0 < b < 1$. Then
(a) f is not invertible on (0, 1) (b) $f \neq f^{-1}$ on (0,1) and $f'(b) = \dfrac{1}{f'(0)}$
(c) $f = f^{-1}$ on (0,1) and $f'(b) = \dfrac{1}{f'(0)}$ (d) f^{-1} is differentiable on (0,1)

50. **(c, d)** $f'(x) = \dfrac{(b^2 - 1)}{(1-bx)^2} < 0 \quad \forall\ x \in (0,1)$ as $b \in (0,1)$
Also, f(x) is continuous for all $x \in (0,1)$ as domain of definition is $R - \{1/b\}$ where $1/b > 1$. Hence, f(x) is strictly decreasing, so invertible.
$$f(x) = y = \frac{b-x}{1-bx} \Rightarrow x = \frac{y-b}{yb-1} \Rightarrow f^{-1}(x) = \frac{b-x}{1-bx} = f(x)$$
$$f'(b) = \frac{1}{b^2 - 1} \text{ and } f'(0) = b^2 - 1$$
Since , $f'(x)$ exist for all $x \in (0,1) \Rightarrow f^{-1}$ is differentiable for all $x \in (0,1)$

51. Let E and F be two independent events. The probability that exactly one of them occurs is 11/25 and the probability of none of them occurring is 2/25. If P(T) denotes the probability of occurrence of the event T, then
(a) $P(E,) = 4/5$, $P(F) = 3/5$ (b) $P(E) = 1/5, P(F) = 2/5$
(c) $P(E) = 2/5$, $P(F) = 1/5$ (d) $P(E) = 3/5, P(F) = 4/5$

51. **(a, d)** $P(E \cap \bar{F}) + P(\bar{E} \cap F) = 11/25$, $P(\bar{E} \cap \bar{F}) = 2/25$
Let P(E) = x, P(F) = y $\therefore P(\bar{E}) \cdot P(\bar{F}) = 2/25 \Rightarrow (1-x)(1-y) = 2/25$
Also $P(E) \cdot P(\bar{F}) + P(\bar{E}) \cdot P(F) = 11/25 \Rightarrow x(1-y) + (1-x)y = 11/25$
Solving x = 3/5 or 4/5 and y = 4/5 or 3/5

52. If $f(x) = \begin{cases} -x-\frac{\pi}{2}, x \le -\frac{\pi}{2} \\ -\cos x,\ -\frac{\pi}{2} < x \le 0, \text{then} \\ x-1,\ 0 < x \le 1 \\ \text{In}x,\ x > 1 \end{cases}$

(a) f(x) is continuous at x = $\pi/2$
(b) f(x) is not differentiable at x = 0
(c) f(x) is differentiable at x = 1
(d) f(x) is differentiable x = -3/2

52. **(abcd)** $f(x) = \begin{cases} -x-\frac{\pi}{2}, x \le -\frac{\pi}{2} \\ -\cos x,\ -\frac{\pi}{2} < x \le 0, \\ x-1,\ 0 < x \le 1 \\ \text{In}x,\ x > 1 \end{cases}$

Continuity at x = $-\pi/2$

L.H.L = R.H.L = $f(-\pi/2) = 0$

$\therefore f'(x) = \begin{cases} -1,\ x \le -\frac{\pi}{2} \\ \sin x,\ -\frac{\pi}{2} < x \le 0 \\ 1,\ 0 < x \le 1 \\ 1/x,\ x > 1 \end{cases}$

Clearly, f(x) is not differentiable at x = 0, f(x) is differentiable at x = 1 and f(x) is differentiable x = -3/2

SECTION – III (TOTAL MARKS: 24)
(INTEGER ANSWER TYPE)

This section contains **6 questions**. The answer to each of the questions is a **single – digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

53. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non – zero complex numbers such that

$$a + b + c = X$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z$$

Then the value of $\dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

53. **(3)** $|x|^2 + |y|^2 + |z|^2 = xx^c + yy^c + zz^c$

$= (a + b + c)$

$\left(a^c + b^c + c^c\right) + \left(a + b\omega + c\omega^2\right)\left(a^c + b^c\omega^2 + c^c\omega\right) + \left(a + b\omega^2 + c\omega\right)\left(a^c + b^c\omega + c^c\omega^2\right)$

$= 3\left(|a|^2 + |b|^2 + |c|^2\right) \Rightarrow \dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2} = 3$

54. Let $y'(x) + y(x)g'(x) = g(x)g'(x),\ y(0) = 0,\ x \in R$, where f'(x) denotes $\dfrac{df(x)}{dx}$ and g(x) is a given non – constant differentiable function on R with g(0) = g(2) = 0. Then the value of y(2) is

54. **(0)** $y'(x) + y(x)g'(x) = g(x)g'(x)$

Linear differential equation with integrating factor $e^{g(x)}$

$\Rightarrow y(x)\cdot e^{g(x)} = \int g(x)\cdot g'(x)\cdot e^{g(x)}dx \Rightarrow y(x)\cdot e^{g(x)} = e^{g(x)}\left(g(x) - 1\right) + c$

Since, $y(0) = 0$ and $g(0) = 0 \Rightarrow c = 1$

$\Rightarrow y(x) = \left(g(x) - 1\right) + e^{-g(x)} \Rightarrow y(2) = \left(g(2) - 1\right) + e^{-g(2)} = 0$

55. Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix},\quad M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix} \text{ and } M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\1\\12\end{bmatrix}$$

Then the sum of the diagonal entries of M is

55. **(9)** $M = \begin{pmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{pmatrix}$

$M\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow x_2 = -1,\ x_5 = 2,\ x_8 = -3$

$M\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \Rightarrow x_1 - x_2 = 1,\ x_4 - x_5 = 1,\ x_7 - x_8 = -1$

$\Rightarrow x_1 = 0,\ x_4 = 3,\ x_7 = -2$

$M\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = M\begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix} \Rightarrow x_1 + x_2 + x_3 = 0;\ x_4 + x_5 + x_6 = 0,\ x_7 + x_8 + x_9 = 12$

$\Rightarrow x_3 = 1,\ x_6 = -5, x_9 = 7$

$\therefore M = \begin{pmatrix} 0 & -1 & 1 \\ 3 & 2 & -5 \\ 2 & 3 & 7 \end{pmatrix}$ $\quad\therefore$ Sum of diagonal entries = 9

56. The straight line 2x – 3y = 1 divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}*$

Then the number of point (s) in S lying inside the smaller part is

56. **(2)** 2x – 3y = 1 meets co – ordinate axes at $\left(\frac{1}{2}, 0\right)$ & $\left(0, \frac{-1}{3}\right) \cdot \left(\frac{5}{2}, \frac{3}{4}\right)$ lies outside the circle hence ruled out.

For, $x = 2, y = \frac{3}{4}$: $2x - 3y - 1 > 0$

For, $x = \frac{1}{4}, y = \frac{-1}{4}$: $2x - 3y - 1 > 0$

For, $x = \frac{1}{8}, y = \frac{1}{4}$: $2x - 3y - 1 < 0$

$\Rightarrow \left(2, \frac{3}{4}\right) \& \left(\frac{1}{4}, \frac{-1}{4}\right)$ are the only two points

57. Let $\vec{a} = -\hat{i} - \hat{k},\ \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then the value of $\vec{r} \cdot \vec{b} = 0$ is

57. **(9)** $(\vec{r}-\vec{c})\times\vec{b}=0 \Rightarrow \vec{r}=\vec{c}+\lambda\vec{b}=(1-\lambda)\hat{j}+(2+\lambda)\hat{j}+3\hat{k}$

If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, then $\vec{r}.\vec{a}=0 \Rightarrow -x-z=0 \Rightarrow x=-z$

Also, $x=1-\lambda,\ y=2+\lambda,\ z=3 \Rightarrow -3=1-\lambda \Rightarrow \lambda=4$

$\therefore \vec{r}=-3\hat{j}+6\hat{j}+3\hat{k}$ $\therefore \vec{r}\cdot\vec{b}=3+6=9$

58. The number of distinct real roots of $x^4-4x^3+12x^2+x-1=0$ is

58. **(2)** Let us assume that all four roots are real and distinct

Hence, f'(x) = 0 must have 3 distinct real roots and

$f''(x)=0$ must have 2 distinct real roots, but that is not true as

$f''(x)=12(x^2-2x+2)$ with D < 0

Hence, f(x) = 0 can't have all four roots real

As, f(0) = -1, f(1) =9 and f(-1) = 15

$\Rightarrow f(x)=0$ must have two distinct roots one in (-1, 0) and the other one in (0, 1)

SECTION – II (TOTAL MARKS: 16)
(MULTIPLE CORRECT ANSWERS TYPE)

This section contains **2 questions**. Each question has four statements (A, B, C and D) given in **Column I** and five statements (p, q , r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with **ONE and MORE** statement(s) given in **Column II**. For example, if for a given question, Statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

59. Match the statements given in Column I with the values given in Column II

	Column I		**Column II**
(A)	If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}, \vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ from a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ be	(p)	$\frac{\pi}{6}$
(B)	If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q)	$\frac{2\pi}{3}$
(C)	The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is	(r)	$\frac{\pi}{3}$
(D)	The maximum value of $\left|\text{Arg}\left(\frac{1}{1-z}\right)\right|$ for $|z| = 1,\ z \neq 1$ is given by	(r)	π
		(t)	$\frac{\pi}{2}$

59. $(A) \to (q), (B) \to (p), (C) \to (s), (D) \to (t)$

For (A): Since, side lengths are 2, 2, $2\sqrt{3}$, hence angle between $\vec{a}$ and $\vec{b}$ is

$$\cos\theta = \frac{4+4-12}{2\cdot 2\cdot 2} = -1/2 \Rightarrow \theta = 2\pi/3$$

For (B) : $\int_a^b f(x)dx = \frac{1}{2}(b^2 - a^2) \Rightarrow f(x) = x \Rightarrow f(\pi/6) = \pi/6$

For (C) : $\frac{\pi}{\ln 3}\left((\ln \sec \pi x + \tan \pi x)\right)_{7/6}^{5/6} = \frac{\pi}{\ln 3}\left[\ln\sqrt{3} - \ln\frac{1}{\sqrt{3}}\right] = \pi$

For (D) : $z = e^{i\theta}$, $\frac{1}{1-z} = \frac{(1-\cos\theta) + i\sin\theta}{(1-\cos\theta)^2 + \sin^2\theta}$

$\therefore \arg\left(\frac{1}{1-z}\right) = \frac{\sin\theta}{1-\cos\theta} = f(\theta)$, which is maximum when $\theta = \pi/2$

60. Match the statements given in Column I with the intervals/ union of intervals given in Column II

	Column I		Column II
(A)	The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right): z \text{ is complex number}, \lvert z\rvert = 1,\ z \neq \pm 1\right\}$	(p)	$(-\infty,-1)\cup(1,\infty)$
(B)	The domain of the function $f(x)=\sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q)	$(-\infty,0)\cup(0,\infty)$
(C)	If $f(\theta)=\begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta): 0\le\theta<\frac{\pi}{2}\right\}$ is	(r)	$[2,\infty)$
(D)	If $f(x)=x^{3/2}(3x-10), x\ge 0$, then f(x) is increasing in	(s)	$(-\infty,-1]\cup[1,\infty)$
		(t)	$(-\infty,0]\cup[2,\infty)$

60. $(A)\to(s)$ Let $k = \text{Re}\left(\frac{2iz}{1-z^2}\right) = \text{Re}\left(\frac{2ie^{i\theta}}{1-e^{2i\theta}}\right) = \text{Re}\left(\frac{2i(\cos\theta+i\sin\theta)}{2\sin^2\theta-2i\sin\theta\cos\theta}\right)$

$= \text{Re}\left(\frac{-2i(\cos\theta+i\sin\theta)}{2i\sin\theta(\cos\theta+i\sin\theta)}\right) = \frac{-1}{\sin\theta}$

It is defined only when $k \in (-\infty,-1]\cup[1,\infty)$

$(B)\to(t)$ Since, $-1\le\frac{8\cdot(3^{x-2})}{1-3^{2x-2}}\le 1$

$\Rightarrow -1\le\frac{8.3^x}{9-3^{2x}}\le 1$

Put $y=3^x$

$\Rightarrow -1\le\frac{8y}{9-y^2}\le 1$

$\therefore\ -1\le\frac{8y}{9-y^2}\Rightarrow\frac{(y-9)(y+1)}{(y+3)(y-3)}\ge 0$

$\Rightarrow\frac{(y-9)}{(y-3)}\ge 0$ ($\because$ y + 1 and y + 3 are always (+) ve)

$\Rightarrow y<3$ and $y\ge 9$

$\Rightarrow x\in(-\infty,1)\cup[2,\infty)$

and $\frac{8y}{9-y^2}\le 1\Rightarrow\frac{(y+9)(y-1)}{(y+3)(y-3)}\ge 0\ \Rightarrow\frac{(y-1)}{(y-3)}\ge 0$

$\Rightarrow y\le 1$ and $y>3$

$\Rightarrow x \in (-\infty, 0] \cup (2, \infty)$

$\therefore$ The common solution is $x \in (-\infty, 0] \cup (2, \infty)$.

$(C) \to (r)$

Since, $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix} = 2\sec^2\theta \therefore f(\theta) \in [2, \infty)$

$(D) \to (r)$ Let $f(x) = x^{3/2}(3x - 10)$

$\Rightarrow f'(x) = (3/2)x^{1/2}(3x - 10) + x^{3/2} \times 3$

For increasing,

$\therefore f'(x) \geq 0$

$\Rightarrow 3\sqrt{x}\left[(3x/2) - 5 + x\right] \geq 0$

$\Rightarrow 3\sqrt{x} \cdot (5x - 10) \geq 0 \Rightarrow x \geq 2$